JOURNAL OF MATERIALS SCIENCE 21 (1986) 3921–3926

The J-integral as a fracture criterion for polycarbonate thermoplastic

R. K. SINGH, K. S. PARIHAR
Defence Materials and Stores Research and Development Establishment, Kanpur 208 013, India

Fracture behaviour of a polycarbonate thermoplastic has been investigated. Fracture tests were conducted on single edge notched specimens and the J-integral evaluated using energy rate interpretation. Its value has been found to be independent of crack length when crack length to specimen width (a/w) is larger than 0.34. For smaller cracks general material damage away from the crack tip is also found to influence the energy absorbed. An extrapolation method has been used to separate the crack tip energy from the energy absorbed due to general material damage. The J-integral thus obtained is independent of crack length and specimen length and its critical value is the same as obtained for $a/w > 0.34$ without extrapolation. The critical stress intensity factor was also evaluated using the R-curve approach. It has been found that the J-integral agrees well with the criticial stress intensity factor obtained using the R-curve approach.

1. Introduction

The rapid development of critical applications of engineering plastics makes it desirable to have practical and reproducible measures of fracture toughness that can be used in design. Most of the research work on fracture of plastic materials has so far centred around the linear elastic fracture mechanics approach employing elastic analysis of the crack tip region. Conventional fracture criteria such as the K_{Ic} or G_{Ic} values, derived from linear elastic fracture mechanics analysis have been successfully applied for relatively brittle polymers such as poly (styrene) and poly (methylmethacrylate) [1, 2]. Linear elastic fracture mechanics cannot be applied when the fracture event being studied occurs at a stress level which is above 70% of the applied stress [3]. However, for ductile polymers such as polycarbonate thermoplastic material, the problem of extensive plasticity at the crack tip has precluded the application of these criteria. The large specimen thicknesses required to induce a plain strain fracture are unsuitable. Moreover the thermoplastic materials such as polycarbonate are predominantly formed in smaller thicknesses (less than 6 mm) and larger thicknesses are unrealistic in terms of the majority of practical applications for these materials. There are also practical difficulties in accurately analysing the crack tip region for homogeneous isotropic materials and more so for heterogeneous materials. The characterization of the crack tip area by a parameter calculated without focusing attention directly at the crack tip would provide a more useful method for analysing fracture. The path independent contour integral (J-integral) proposed by Rice and others [4–7] is such a parameter. The value depends on the near tip stress–strain field. However, the path independent nature of the integral allows an integration path, taken sufficiently far away from the crack tip, to be substituted for a path close to the crack tip region. Therefore, the J-integral can be calculated using numerical methods more accurately compared to the stress intensity factor. The J-integral can be evaluated experimentally quite easily by considering the load–deflection curves of identical specimens with varying crack lengths.

The use of the J-integral as an elastic–plastic fracture criterion has been discussed by Broberg [8] from an analytical point of view. A justification for choosing this parameter as a fracture criterion comes from the consideration of the Hutchinson–Rice–Rosengren (HRR) crack tip model [5, 9], where the product of plastic stress and strain is shown to have $1/r$ singularity where r is a near tip crack field length parameter. For a deformation plasticity theory McClintock [6] has demonstrated, through the crack tip plastic stress and strain equations expressed from HRR singularity, the existance of a singularity in r whose strength is the J-integral. In this way, the J-integral may be chosen as a parameter for the characterization of the crack tip environment because it can be evaluated experimentally and calculated with less difficulty than the plastic stress and strain intensity factors. Begley and Landes [10–12] discussed various aspects of fracture for metals using the J-integral. They demonstrated the applicability of the J-integral for the case of large scale plasticity at the tip of the crack through experimental results on an intermediate strength rotor steel for which the J-integral at failure for fully plastic behaviour was found to be equal to the linear elastic value of the strain energy release rate (G) at failure for extremely large specimens. Thus the J-integral approach eliminates the necessity of testing very large specimens.

Agarwal *et al.* [13] have applied the J-integral as a fracture criterion for composite materials. Williams [14] has discussed the J-integral testing of polymers in detail and Hodgkinson and Williams [15] have applied it in fracture studies of low density polyethylene. Sridharan and Broutman [16] have also used SEN tension specimens for fracture toughness studies of acrylonitrile–butadiene styrene (ABS) by the J-integral method.

One of the most important limitations in the approach is that the J-integral is path independent only when the stress–strain relation is unique. It is truly path independent for linear and non-linear elastic stress–strain laws and also for elastic–plastic behaviour under situations of monotonic loading. The kinetic energy of the molecules and the heat transfer effects are some more limitations which are neglected during the application of the J-integral.

In the present paper, the J-integral is being developed as a fracture criterion for polycarbonate thermoplastics based on test results. An extrapolation method has been used to separate the crack tip energy from the energy absorbed due to general material damage. The results were compared with the critical stress intensity factor obtained experimentally by using the R-curve approach.

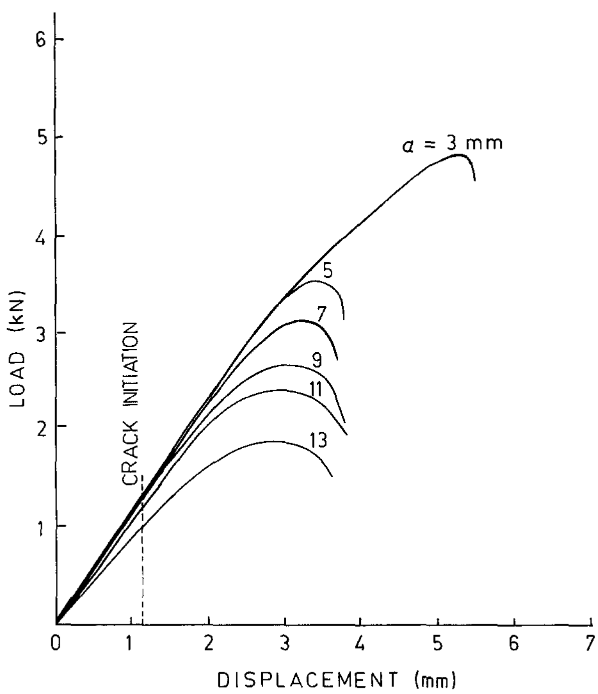

Figure 1 Load–displacement curves for different initial crack length.

2. Experimental details

The present studies were performed on polycarbonate (Makrolon) thermoplastic extruded sheet 4 mm thick (Bayer, West Germany). The single edge notched tension specimens (25 mm wide) were used for measuring fracture toughness and the length between grips was at least three times the specimen width. Cracks of lengths 3, 5, 7, 9, 11 and 13 mm were machined in the specimens by using a 0.2 mm thick cutter on the milling machine to study fracture toughness with the use of the J-integral and the R-curve approach. Cracks in some samples were further cut by forcing a razor blade into the material slowly to a depth of 0.5 mm for cross checking the value of fracture toughness at instability. The fracture toughness tests were performed on universal strength testing machine FPZ-10. Load and load point displacement were recorded on an x–y recorder. All the tests were conducted in a displacement controlled mode. The data were analysed using the J-integral approach. The fracture toughness tests were also performed on 10 tonne Tenius Olson universal testing machine. Load and crack mouth opening displacement (COD) were recorded on an x–y recorder. All the tests were performed at a crosshead speed of 0.5 mm min^{-1}, in an airconditioned laboratory (temperature 25 $\pm$ 0.5° C; relative humidity 50 $\pm$ 5%). Load–crack mouth opening displacement was analysed using the R-curve approach.

3. Results and discussion

Typical load–displacement (at load point) curves for specimens with different initial crack lengths are shown in Fig. 1. The tests were conducted under the displacement controlled conditions so that the load–displacement curves beyond maximum load are also indicated. Specimens with smaller crack lengths fracture suddenly causing drops in load, whereas the specimens with larger crack lengths show a more gradual fracture process beyond the maximum load. This is because strain energy stored during loading in specimens having small crack lengths is sufficient to cause catastrophic failure. It is not the case with the specimens having longer crack lengths. It has been observed that the initial crack begins to propagate at a displacement of 1.15 mm (Fig. 1).

The nature of load–displacement curves show that a considerable amount of slow stable ductile crack growth occurs before the final catastrophic fracture. The specimens tested showed ductile tearing with the formation of shear lips on the fracture faces. When ductile tearing does occur, it is usually under plane stress conditions, although initiation is frequently in plain strain with a rather rapid transition to plane stress with gross lateral contraction. There is little experience to draw on for the use of fracture mechanics in this sort of situation, but in principle it is possible to use the J-integral as a fracture criterion. The points on the load–displacement curve, where the fracture process becomes unstable (i.e. instability point) at a displacement beyond which the load decreases monotonically is referred as critical displacement. This is plotted against initial crack length as shown in Fig. 2. Initially the critical displacement decreases with increase of initial crack length and remains constant for cracks larger than 8.50 mm. The initial variation in critical displacement occurs due to large loads. The critical value of the J-integral is obtained corresponding to the constant critical displacement of 3 mm as shown in Fig. 2.

The load–displacement curves can be used to obtain the value of the J-integral experimentally through its energy interpretation as follows

$$J = -\left(\frac{\partial U}{\partial a}\right) \quad \text{constant displacement} \quad (1)$$

where U is the potential energy per unit thickness and a is the crack length.

It may be mentioned that the displacement is kept

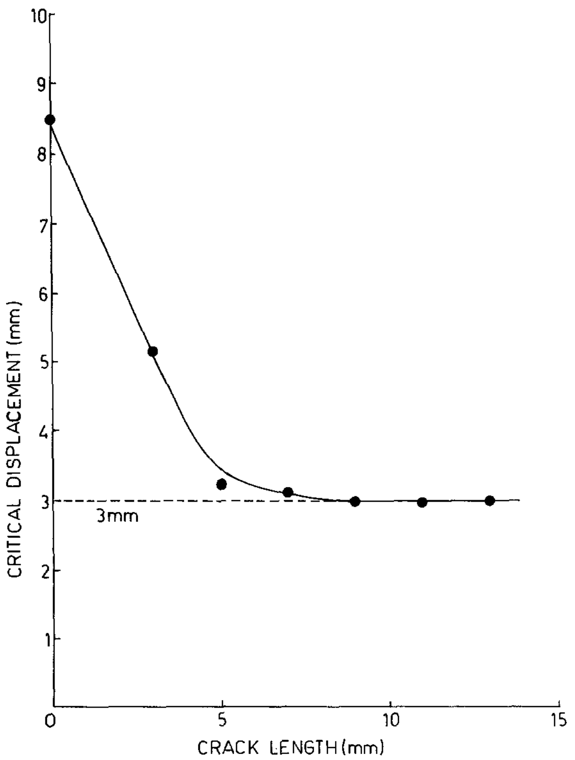

Figure 2 Variation of critical displacement with initial crack lengths.

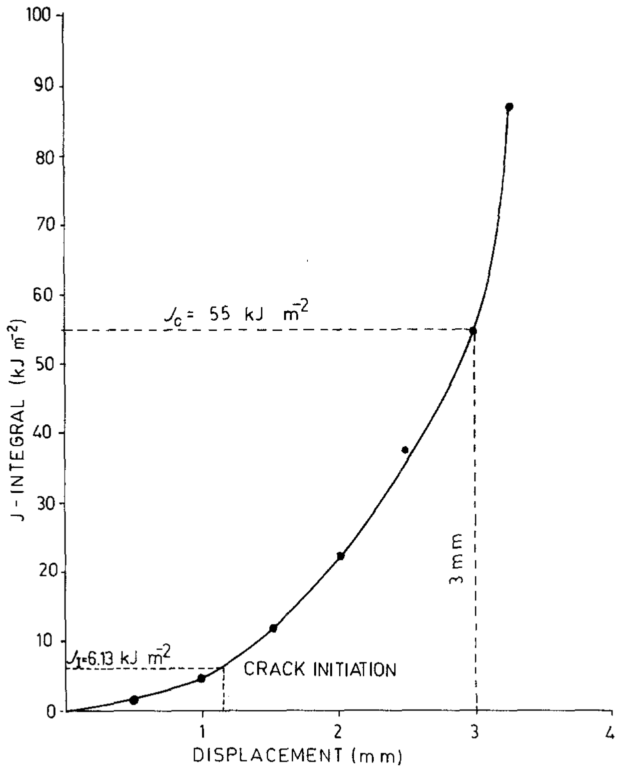

Figure 4 J-Integral as a function of displacement.

constant to evaluate the J-integral. Thus the potential energy U reduces the area under the load–displacement curve and is equal to the strain energy [11]. Therefore, the strain energy is obtained from the area of the load–displacement curves for several displacements. From the experimental data a plot of strain energy, U, as a function of initial crack length is plotted. One curve is obtained for each value of displacement. A family of such curves for SEN tension specimen is shown in Fig. 3. Similar type of curves were also

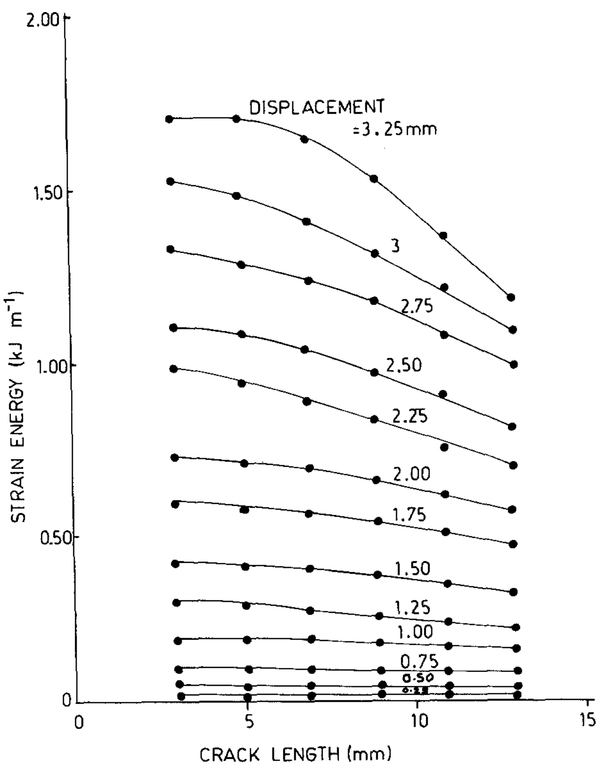

Figure 3 Strain energy per unit thickness of specimens for different displacement.

obtained by Sridharan and Broutman [16] during fracture toughness studies of ABS plastics by the J-integral.

For a given displacement, energy absorbed by a specimen decreases as the crack length increases (Fig. 3) because smaller loads are required. The variation in energy absorbed is less for cracks smaller than 8.50 mm as compared to that for larger cracks, because in specimens with larger cracks, the energy absorbed is essentially in the vicinity of the crack tip and thus strongly influenced by the crack length.

The J-integral is obtained from Equation 1 through the slopes of the energy curves as shown in Fig. 3. The J-integral is independent of crack length for cracks larger than 8.50 mm, as the energy curves are straight lines in this range (Fig. 3).

The variation of J-integral with displacement is shown in Fig. 4., The critical value of J corresponding to the critical displacement of 3 mm is 55 kJ m^{-2}.

The critical value of J_c 55 kJ m^{-2} is a plane stress value corresponding to a crack growth of about 4 mm (determined from the R-curve of initial crack length of 3 mm). Ferguson *et al.* [17] have reported the value of J_c as 49.1 kJ m^{-2} at maximum load for the same material. The value of J_c obtained in this experiment is some higher than the value reported by the above authors. The reason for higher value may be due to the following:

(a) The slow stable ductile crack occurs during loading in plane stress conditions with the load maximizing and subsequently falling. The increased load involved leads to extensive plasticity and subsequently non-linearity in the load–deflection curve.

(b) The initial crack may be machined blunt, however, in some samples the cracks were cut by forcing a razor blade to get sharp cracks in order to check the value of J_c obtained, but no significant variation has been observed.

(c) The induced bending in SEN tension causes compressive yielding at the back which may lead higher value of J_c, but this effect has been minimized by keeping the symmetry during the testing. As such this has negligible effect on the value of J_c.

For smaller cracks, the value of J-integral depends upon the displacement as well as crack length because the slope of the energy curve changes with crack length (Fig. 3). The variation of J-integral for cracks smaller than 8.50 mm has not been shown because it is not unique. However, Fig. 3 shows that in this region the value of the J-integral will be smaller for a given displacement but defined for a greater range of displacement. Its apparent critical value is also expected to be larger in these cases.

From the preceding paragraph it is observed that when the crack is larger than 8.50 mm or when $a/w > 0.34$, the fracture behaviour is governed essentially by the crack tip environment resulting in a constant critical displacement and a unique value of the J-integral. For these crack lengths the fracture load is small which does not cause any general material damage away from the crack tip region. On the other hand when cracks are small (< 8.50 mm or $a/w < 0.34$), the J-integral and critical displacement depend on the crack length, indicating that in addition to the crack tip environment, the region away from it also influences such qualities as the energy absorbed and displacement of fracture. This is due to the high fracture loads which cause general material damage.

In order to study the influence of general material damage in specimens with smaller cracks three specimens of 7 mm, 9 mm and 11 mm crack lengths with varying specimen lengths were tested. The length between grips was varied from 3 to 5 times the width of the specimen. The critical displacement (displacement at fracture) is plotted against specimen length in Fig. 5. The critical displacement was found to increase with the increase of specimen length for all the crack lengths, whereas this was found nearly the same for specimens with cracks 9 and 11 mm long, which is consistent with Fig. 2. The total displacement of the specimen is the sum of the displacement in the crack tip region, which may be expected to be independent of the specimen length and displacement in the region away from the crack tip which should be a function of specimen length. The intercept on the ordinate obtained through extrapolation of the straight line in Fig. 5 is considered as the displacement due crack tip region alone. All the straight lines in Fig. 5 intercept the ordinate at the same point. This common intercepting point may be regarded as a critical displacement due to the presence of the crack and whose value is independent of crack length as well as specimen length.

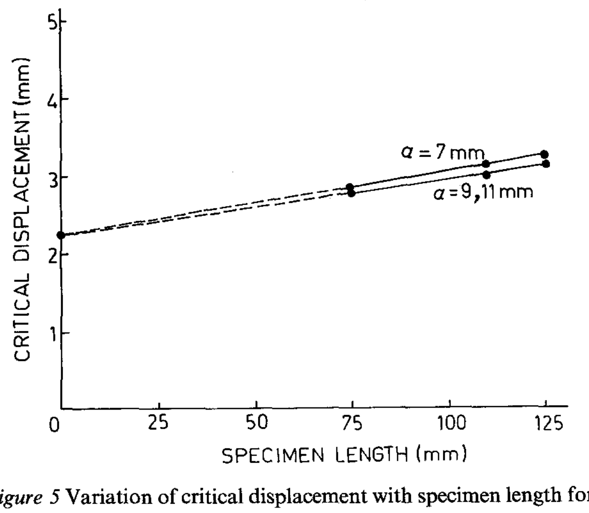

Figure 5 Variation of critical displacement with specimen length for different crack lengths.

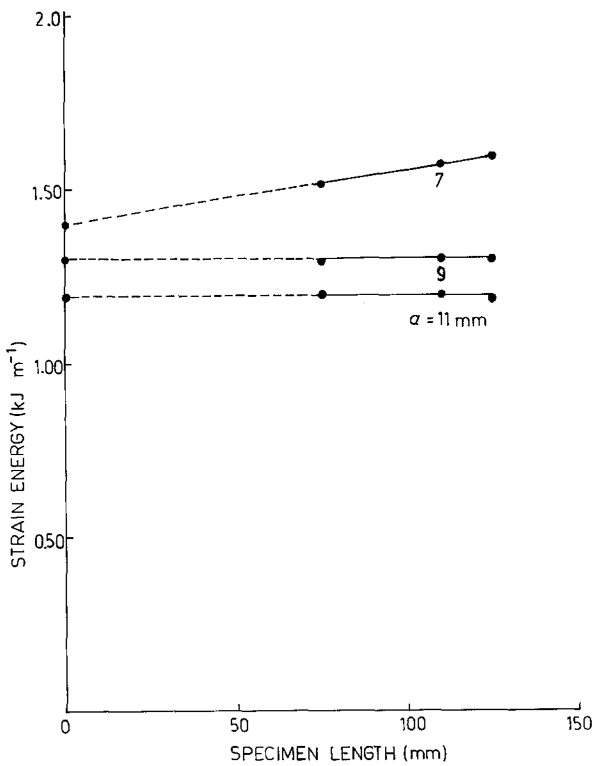

Figure 6 Variation of strain energy with specimen length for different crack lengths.

Variation of energy absorbed up to fracture for different crack lengths are plotted against specimen lengths in Fig. 6. The total energy absorbed may be considered as the sum of the energies absorbed in the crack tip region and the region away from it. The energy absorbed in the crack tip region depends on the crack length and not on the specimen lengths, whereas the energy absorbed in the region away from the crack tip depends upon the specimen length (Fig. 6). It is observed from Fig. 6 that when the crack length is 9 or 11 mm the energy absorbed is independent of the specimen length, indicating negligible energy absorption in the region away from the crack tip and the damage is mainly confined to the crack tip region. For crack length of 7 mm, the total energy absorbed increases linearly with the specimen length indicating a significant energy absorption in the region away from the crack tip and the damage is all over the specimen length.

The intercept on the ordinate obtained by extrapolation of straight line in Fig. 6 is the energy absorbed in the crack tip region alone. This energy absorbed is plotted in Fig. 7. It has already been pointed out that the critical displacement due to the introduction of the crack alone is independent of crack length. Therefore,

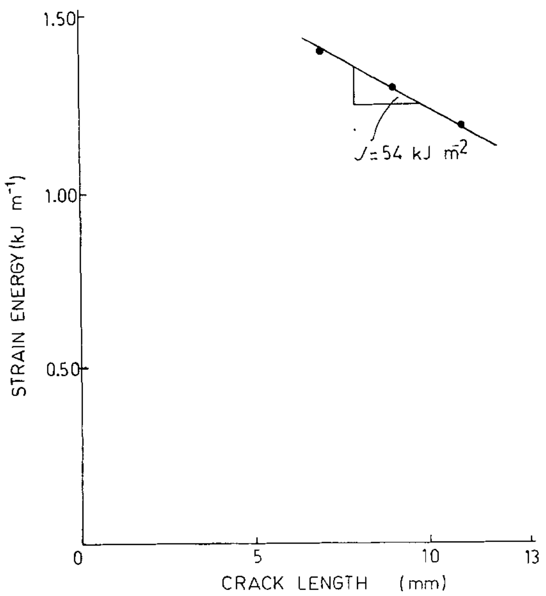

Figure 7 Strain energy at the crack tip for different crack lengths.

the energy absorbed for different crack lengths (Fig. 7) is considered to correspond to the same critical displacement and as such the slope of the straight line may be used to obtain the critical value of the J-integral independent of crack length. The critical value of the J-integral (J_c) thus obtained is equal to $54\,kJ\,m^{-2}$ which is close to the value $55\,kJ\,m^{-2}$ obtained earlier in Fig. 4. This shows that the energy absorbed at the crack tip may be isolated from that energy absorbed away from it. Thus a parameter independent of testing variables (i.e crack length and specimen length) is obtained which may be used as a fracture criterion for the material.

The load–displacement (COD) records were analysed in accordance with the procedure recommended in ASTM-E-399-71. The stress intensity factor K_I was calculated by using the following relationship:

$$K_I = \frac{YP(a)^{1/2}}{tw} \qquad (2)$$

where P is the applied load, a is the crack length, w the specimen width, t the thickness of specimen and Y is the calibration fator. Its value is taken same as that for isotropic materials [18]. The same value of calibration factor has also been used by Williams [19].

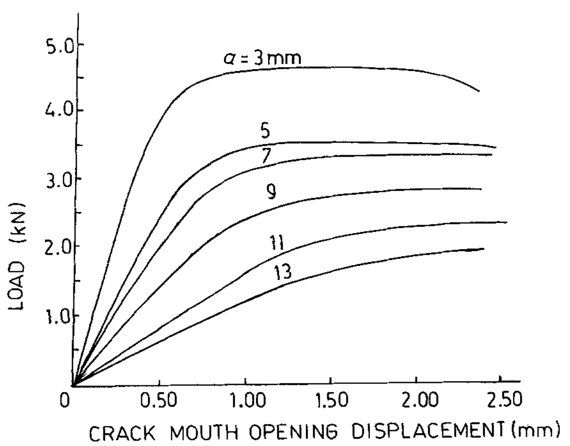

Figure 8 Load against crack mouth opening displacement (COD) for different initial crack lengths.

$$Y = 1.99 - 0.41(a/w) + 18.7(a/w)^2 - 38.48(a/w)^3 + 53.85(a/w)^4 \qquad (3)$$

The load–crack mouth opening displacement (COD) curves are shown in Fig. 8 for different crack lengths. The load–displacement curves are initially linear but deviate pregressively from linearity because of appreciable damage ahead of the crack tip. The nature of these curves suggests that considerable amount of slow crack growth occurs in this material before the final catastrophic fracture. The crack tip damage is neither colinear nor coplanar with the initial crack. This damage zone surrounding the crack tip is considered to be the plastic zone. As the damage at the crack tip increases, the compliance of the specimen also increases.

For the sake of analysis, the damage growth of the crack tip can be approximately as self similar crack extension using compliance matching as proposed by Gagger and Broutman [20] for random fibre composites. The initial compliances were obtained from load–COD curves and plotted with the ratio of initial crack length to width of specimen as shown in Fig. 9. The instantaneous compliances during the fracture toughness test were calculated from load–COD curves. The effective crack length corresponding to instantaneous compliances were obtained from the crack length estimation curves (Fig. 9). Stress intensity factor (K_I) was calculated from Equation 2 for loads corresponding to instantaneous compliances and effective crack length (a). This value of the stress intensity factor is the instantaneous crack growth resistance (K_R). K_R values are obtained for increasing loads up to fracture. Crack growth resistance curves (R-curves) were plotted from K_R values and effective crack lengths for different initial crack lengths up to 7 mm (Fig. 10). However, it is shown for the 7 mm of crack length only. R-curves for crack lengths of 9 mm and

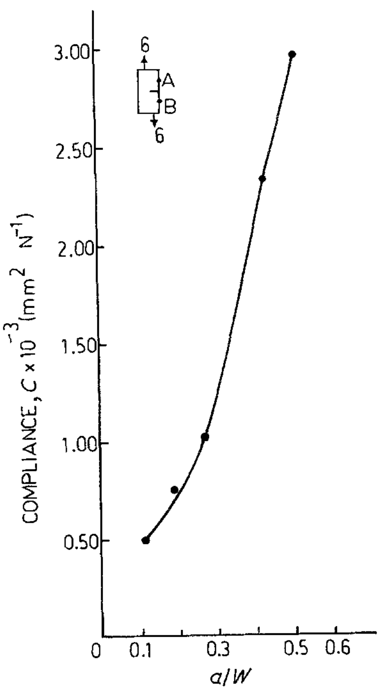

Figure 9 Crack length estimation curve. $c = \delta \times t/p$; δ = displacement between A and B; t = thickness; P = load.

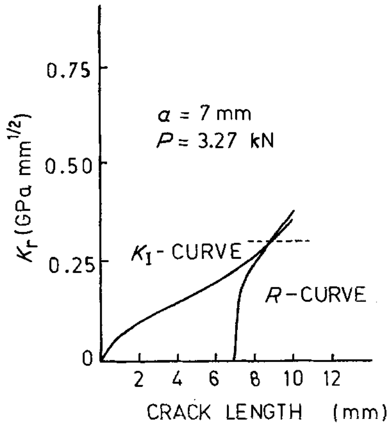

Figure 10 Determination of K_c through the R-curve for 7 mm initial crack length. Point of tangency, $K_c = 0.33$ GPa (mm)$^{1/2}$ (10.60 MPa m$^{1/2}$).

above could not be obtained as the effective crack lengths at fracture become more than 13 mm.

Critical crack growth resistance, K_R (instability) or the critical stress intensity factor, K_c (for plane stress) were obtained from the R-curve and stress intensity factor K_I curves for constant load (near the fracture load) plotted against crack lengths on the same graph. The tangent point of the R-curve and K_I curve gives the value of critical stress intensity factor K_c equal to 10.60 MPa m$^{1/2}$ (critical crack growth resistance). This is shown in Fig. 10.

For the plane stress case, the critical value of the J-integral (J_c) is related to critical stress intensity factor (K_c) in mode I deformation by the following relationship [21]

$$J_c = \frac{K_c^2}{E} \tag{4}$$

where E is the modulus of elasticity of the material. The present material has an average modulus of elasticity E equal to 2.12 GPa. Therefore by substituting the value of J_c in Equation 4 we get K_c equal to 10.75 MPa m$^{1/2}$. This value of critical stress intensity factor agrees very well with the K_c of 10.60 MPa m$^{1/2}$ obtained above from the R-curve analysis. The value of the J-integral at the point of crack initiation is found to be 6.13 kJ m^{-2}. This demonstrates that the J-integral method of characterizing fracture toughness is consistent with the R-curve approach. However, the J-integral method is a lot simpler for experimental as well as analytical (computational) evaluation.

4. Conclusions

Fracture behaviour of polycarbonate thermoplastic material has been investigated. The J-integral has been evaluated using the energy interpretation. Its value is found to be independent of crack length when the ratio of crack length to specimen width (a/w) is larger than 0.34. For smaller crack lengths general material damage away from the crack tip also influences the energy absorbed. An extrapolation method has been used to separate crack tip energy from the energy absorbed due to general material damage. The J-integral thus obtained is independent of crack length and specimen length and its critical value is the same as obtained for $a/w > 0.34$ without extrapolation. The critical stress intensity factor was also evaluated using the R-curve approach. The J-integral agrees well with the critical stress intensity factor obtained using the R-curve analysis.

References

1. L. J. BROUTMAN and T. KOBAYASHI, AMMRC CR 71-14, Army Materials Mechanical Research Centre, Water Town, Massachussetts (1971).
2. C. S. LEE, PhD thesis, University Michigan (1974).
3. H. W. LIU, *ASTM STP* **381** (1965) 23.
4. J. R. RICE, *J. Appl. Mech.* **35** (1968) 379.
5. J. R. RICE and G. F. ROGENGREN, *J. Mech. Phys. Solids* **16** (1968) 1.
6. F. McCLINTOCK, "Plasticity Aspects of Fracture", Vol. III edited by H. Liebowitz (Academic Press, New York, 1971) pp. 47–225.
7. J. R. RICE, P. C. PARIS and J. G. MERKLE, ASTM STP 536 (American Society for Testing and Materials, 1973) pp. 231–245.
8. K. B. BROBERG, *J. Mech. Phys. Solids* **19** (1971) 407.
9. J. W. HUTCHINSON, *ibid.* **16** (1968) 13.
10. J. A. BEGLEY and J. D. LANDES, ASTM STP 514 (American Society for Testing and Materials, 1972) pp. 1–20.
11. J. D. LANDES and J. A. BEGLEY, ASTM STP 514, (American Society for Testing and Materials, 1972) pp. 24–39.
12. J. D. LANDES and J. A. BEGLEY, ASTM STP 632, (American Society for Testing and Materials, 1977) pp. 57–81.
13. B. D. AGARWAL, K. PRASHANT and B. S. PATRO, in Proceedings of the 2nd International Conference on Composite Structures, Scotland, 14–16 Sept, 1983, edited by I. H. Marshall (Applied Science, London, 1983) pp. 486–499.
14. J. G. WILLIAMS, in "Fracture Mechanics of Polymers" (Ellis Horwood, Chichester, 1984) 1st Edn, pp. 132–142.
15. J. M. HODGKINSON and J. G. WILLIAMS, *J. Mater. Sci.* **16** (1981) 50.
16. N. S. SRIDHARAN and L. J. BROUTMAN, *Polym. Eng. Sci.* **22** (1982) 760.
17. R. J. FERGUSON, G. P. MARSHALL and J. G. WILLIAMS, *Polymer* **14** (1973) 451.
18. W. F. BROWN and J. E. SRAWLEY, ASTM STP 410 (American Society for Testing and Materials, 1968).
19. J. G. WILLIAMS, in "Fracture Mechanics of Polymers" (Ellis Horwood, Chichester, 1984) 66.
20. S. K. GAGGER and L. J. BROUTMAN, *J. Composite Mater.* **9** (1975) 216.
21. J. R. RICE, in "Mathematicel Analysis in the Mechanics of Fracture in Fracture", Vol. II, edited by H. H. Liebueitz. (Academic Press, New York, 1968) pp. 191–311.

Received 15 July 1985
and accepted 21 January 1986